

# TERRAIN-BASED ROBOT NAVIGATION USING MULTI-SCALE TRAVERSABILITY INDICES

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## Abstract

*The concepts of Local, Regional, and Global Traversability Indices have been introduced recently [1, 8-9]. These indices represent the suitability of a terrain for traversal by a mobile robot at different scales of resolution. This paper utilizes these indices to develop a navigation strategy for a mobile robot traversing a challenging terrain. The traversability indices form the basis of three navigational behaviors; namely, Traverse-Local, Traverse-Regional, and Traverse-Global behaviors. These behaviors are blended with the Seek-Goal behavior to ensure that the mobile robot reaches the goal safely while avoiding obstacles and impassable terrain segments. The paper is concluded by an illustrative graphical simulation study*<sup>1</sup>.

## 1 Introduction

Exploration of planetary surfaces and operation in rough terrestrial terrain have been strong motivations for research in autonomous navigation of field mobile robots in recent years. These robots must cope with two fundamental problems. The first problem is to acquire and analyze the terrain quality information on-line and in real time, and to utilize it in conjunction with limited prior terrain imagery. The second problem is to deal with imprecision in sensor measurement and uncertainty in data interpre-

tation inherent in sensing and perception of natural environments. Because of these two fundamental problems, outdoor robot navigation defines a new research topic that is distinct from the conventional indoor robot navigation in structured and benign man-made environments.

Robust on-line terrain characterization and traversability assessment are clearly core research problems for autonomous field robot navigation. Two types of solutions have been proposed to date by researchers at CMU and JPL. In the CMU methods [2-7], the robot traversability is computed along different arcs that correspond to different steering angles. The traversability of each arc is determined mathematically by a weighted sum of the roll, pitch, and roughness of the map cells along that arc, incorporating their certainty values [2]. The JPL approach [8-13] takes a sharp departure from analytical methods and is centered on the *Rule-Based Traversability Index*. This index is a novel concept that was first introduced in [8-9] as a simple *linguistic* measure for quantifying the suitability of the regional terrain for traversal by a mobile robot. This concept was extended subsequently to local and global terrain traversability [1]. This perceptual approach to terrain assessment is highly robust to measurement noises and interpretation errors because of the use of fuzzy sets in a linguistic rule-based system. This approach is analogous to the human judgment, reasoning, and decision-making regarding assessment and traversal of a natural terrain.

The paper is structured as follows. Section 2 reviews terrain traversability indices at local, regional, and global scales. The navigation behaviors based on these measures are presented in Section 3. Section 4 discusses the integration of multiple behavior

<sup>1</sup>The research described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Thanks are due to Bruce Bon of JPL for developing the Robot Graphical Simulator software used in Section 5.

recommendations for robot navigation. An illustrative simulation study is presented in Section 5. The paper is concluded in Section 6 with a review of key features and areas of future research.

## 2 Review of Traversability Indices

The concepts of Local, Regional, and Global Traversability Indices have been introduced recently [1, 8-9]. These indices represent the suitability of a terrain for traversal by a mobile robot at different scales of resolution. The Local Traversability Index is related by a set of rules to local obstacles and surface softness, measured by on-board sensors mounted on the robot. The rule-based Regional Traversability Index is computed from the terrain roughness and slope that are extracted from video images obtained by on-board cameras. The Global Traversability Index is obtained from the terrain topographic map and is based on the natural or man-made surface features such as mountains and craters. Each traversability index is represented by four fuzzy sets with the linguistic labels {POOR, LOW, MODERATE, HIGH}, corresponding to surfaces that are unsafe, moderately-unsafe, moderately-safe, or safe for traversal, respectively.

## 3 Terrain-Based and Goal-Based Navigation Behaviors

The control variables of the mobile robot are the translational speed  $v$  and the rotational speed (or turn rate)  $\omega$ , where  $v = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}$ ,  $\omega = \frac{d\theta}{dt}$ , and  $x$ ,  $y$ , and  $\theta$  are the position coordinates of the robot center and the robot orientation in the reference frame, respectively. The robot speed  $v$  is represented by the four linguistic fuzzy sets {STOP, SLOW, MEDIUM, FAST}. Similarly, the robot turn rate  $\omega$  is represented by the three linguistic fuzzy sets {LEFT, ON - COURSE, RIGHT}.

We shall now describe three terrain-based and one goal-based behaviors that constitute the robot navigation system.

### 3.1 Traverse-Terrain Behaviors

Three independent behaviors can be defined based on the three traversability analyses; namely, traverse-local behavior, traverse-regional behavior, and traverse-global behavior. These behaviors issue motion recommendations to the robot control system on the basis of the local terrain, regional terrain, and global terrain quality information, respectively. We shall now use the Traversability Indices reviewed in Section 2 to develop simple rules for determination of the robot motion (i.e., turn rate and speed) while moving on natural terrain. It is assumed that the robot can only move in the forward direction (i.e., reverse motion is not allowed). The terrain in front of the robot is partitioned into three groups of 60° circular sectors as shown in Figure 1, namely: front, right, and left of the robot. When higher resolution is needed, the 180° field-of-view can be decomposed into a larger number of smaller sectors and similar navigation rules can be developed. The local, regional, and global circular sectors have the radii  $R_l$ ,  $R_r$ , and  $R_g$ , respectively. The Traversability Indices for the three regions,  $\tau^f$ ,  $\tau^r$ ,  $\tau^l$ , are assumed to be available from sensory data (for local and regional behaviors) or from the traversability map (for global behavior), as described in [1].

We shall now develop a set of rules for robot motion based on the terrain traversability indices. We treat local, regional, and global traversability indices in a unified manner by describing a “universal” set of rules for all three terrain-based navigation behaviors. These three behaviors are separated out in the blending section later.

#### 3.1.1 Turn Rules

The terrain-based turn rules are summarized in Table 1. These rules represent the steering actions of a human driver during an off-road driving session. Observe that a turn maneuver is not initiated when either the front region is the most traversable, or the right and left regions have the same traversability indices as the front region. Observe that the “preferred” direction of turn is chosen arbitrarily to be LEFT, i.e., when the robot needs to turn to face a more traversable region, it tends to turn left.

### 3.1.2 Move Rules

Once the direction of traverse is chosen based on the relative values of  $\tau$ , the robot speed  $v$  can be determined based on the value  $\tau^*$  of the Traversability Index  $\tau$  in the *chosen direction*. This determination is formulated as a set of simple fuzzy logic rules for speed of traverse as follows:

- IF  $\tau^*$  is POOR, THEN  $v$  is STOP.
- IF  $\tau^*$  is LOW, THEN  $v$  is SLOW.
- IF  $\tau^*$  is MODERATE, THEN  $v$  is MEDIUM.
- IF  $\tau^*$  is HIGH, THEN  $v$  is FAST.

This is analogous to a human driver adjusting the car speed based on the surface conditions.

## 3.2 Seek-Goal Behavior

The seek-goal behavior is a *map-based, deliberative* behavior whose objective is to navigate the mobile robot from a known initial position to a user-specified goal position disregarding the terrain quality. Both positions are expressed by the  $\{x, y\}$  coordinates with respect to a fixed map-based frame-of-reference.

In this section, we present a set of fuzzy logic navigation rules for the seek-goal behavior. In these rules, the robot initially performs an in-place rotation toward the goal to nullify the heading error. Once the robot is aligned with the goal direction, it then proceeds toward the goal position. A similar rule set can also be formulated for robots that cannot perform in-place rotation.

### 3.2.1 Turn Rules

The fuzzy logic rules for the robot rotational motion are as follows:

- IF  $\phi$  is LEFT, THEN  $\omega$  is LEFT.
- IF  $\phi$  is HEAD-ON, THEN  $\omega$  is ON-COURSE.
- IF  $\phi$  is RIGHT, THEN  $\omega$  is RIGHT.

where the heading error  $\phi$  is represented by the three linguistic fuzzy sets {LEFT, HEAD-ON, RIGHT}.

### 3.2.2 Move Rules

The following rules are used for the robot translational motion:

- IF  $d$  is VERY-NEAR OR  $\phi$  is NOT HEAD-ON, THEN  $v$  is STOP.
- IF  $d$  is NEAR AND  $\phi$  is HEAD-ON, THEN  $v$  is SLOW.
- IF  $d$  is FAR AND  $\phi$  is HEAD-ON, THEN  $v$  is MEDIUM.
- IF  $d$  is VERY-FAR AND  $\phi$  is HEAD-ON, THEN  $v$  is FAST

where the position error (goal distance)  $d$  is represented by the four linguistic fuzzy sets {VERY-NEAR, NEAR, FAR, VERY-FAR}. Rule 1 keeps the robot stationary while it is correcting its heading. In rules 2-4, the robot is aligned with the goal direction and moves with a speed proportional to its distance from the goal.

## 4 Integration of Multiple Behaviors

In this section, we blend navigational recommendations from the two *sensor-based, reactive* behaviors (traverse-local and traverse-regional) with the two *map-based, deliberative* behaviors (traverse-global and seek-goal).

Let the weighting factors  $l^w$ ,  $r^w$ ,  $g^w$ , and  $s^w$  represent the strengths by which the traverse-local, traverse-regional, traverse-global, and seek-goal recommendations are taken into account to compute the final control actions  $\bar{v}$  and  $\bar{\omega}$ . These weights are represented by the three linguistic fuzzy sets {LOW, NOMINAL, HIGH}. Four sets of weight rules for the four behaviors are now presented.

The seek-goal weight rules are as follows:

- IF  $d$  is VERY-NEAR, THEN  $s^w$  is HIGH.
- IF  $d$  is NOT VERY-NEAR, THEN  $s^w$  is NOMINAL.
- IF  $\tau_f^l$  is POOR, THEN  $s^w$  is LOW.

The traverse-local weight rules are as follows:

- IF  $d$  is NOT VERY-NEAR, THEN  $l^w$  is HIGH.
- IF  $d$  is VERY-NEAR, THEN  $l^w$  is NOMINAL.

The traverse-regional weight rules are as follows:

- IF  $d$  is NOT VERY-NEAR AND  $\tau_f^l$  is NOT POOR, THEN  $r^w$  is HIGH.
- IF  $d$  is VERY-NEAR OR  $\tau_f^l$  is POOR, THEN  $r^w$  is NOMINAL.

The traverse-global weight rules are as follows:

- IF  $d$  is NOT VERY-NEAR AND  $\tau_f^l$  is NOT POOR AND  $\tau_f^r$  is NOT POOR, THEN  $g^w$  is HIGH.
- IF  $d$  is VERY-NEAR OR  $\tau_f^l$  is POOR OR  $\tau_f^r$  is POOR, THEN  $g^w$  is NOMINAL.

At each control cycle, the above sets of weight rules are used to calculate the four crisp weighting factors using the Center-of-Gravity (Centroid) defuzzification method [14]. The motion recommendations from the seek-goal, traverse-local, traverse-regional, and traverse-global behaviors are then weighted by the corresponding gains  $s^w$ ,  $l^w$ ,  $r^w$ , and  $g^w$  respectively prior to defuzzification, as shown in Figure 2. The final control actions are computed using the Center-of-Gravity defuzzification method as:

$$\bar{v} = \frac{s^w \Sigma v_p^s \cdot A_p^s + l^w \Sigma v_p^l \cdot A_p^l + r^w \Sigma v_p^r \cdot A_p^r + g^w \Sigma v_p^g \cdot A_p^g}{s^w \Sigma A_p^s + l^w \Sigma A_p^l + r^w \Sigma A_p^r + g^w \Sigma A_p^g}$$

$$\bar{\omega} = \frac{s^w \Sigma \omega_p^s \cdot B_p^s + l^w \Sigma \omega_p^l \cdot B_p^l + r^w \Sigma \omega_p^r \cdot B_p^r + g^w \Sigma \omega_p^g \cdot B_p^g}{s^w \Sigma B_p^s + l^w \Sigma B_p^l + r^w \Sigma B_p^r + g^w \Sigma B_p^g}$$

In the above equations,  $v_p$  and  $A_p$  are the peak membership value and the truncated area under the membership function for the velocity fuzzy sets, while  $\omega_p$  and  $B_p$  are the corresponding values for the turn rate fuzzy sets.

## 5 Simulation Study

The Robot Graphical Simulator software package is developed at JPL for visualization of the robot motion using the reasoning and decision-making capabilities of the fuzzy logic navigation strategy. The map of the terrain on which the robot moves is

available *a priori* and is converted to a Global Traversability Map [1] in which terrain traversability is graded using the four linguistic fuzzy sets {POOR, LOW, MODERATE, HIGH}. In the simulation study, the terrain is composed of the following two types of surfaces:

- *Plain*: Land area having a level surface with no major obstacles and HIGH Traversability Index.
- *Impassable Regions*: Regions of high rock concentration, crater, or steep slope and POOR Traversability Index. The locations of these regions are known *a priori* to the mobile robot through the previously-acquired Global Traversability Map.

In addition, there are several large rocks on the terrain. The rock locations are *not* known *a priori* to the mobile robot. These rocks are detected by on-board proximity sensors. These sensors are modeled in the software by computing the distance between the robot and each rock.

There are two large rocks, a crater with POOR Traversability Index, and a region of high rock density with POOR Traversability Index between the initial and the goal positions of the robot, as depicted in Figure 3. The robot is required to drive to the goal position while avoiding both rocks and both impassable regions. The Traverse-Local, Traverse-Global, and Seek-Goal behaviors are invoked to guide the robot safely to its destination. The path traversed by the robot under the fuzzy logic navigation rules is shown by the dotted line in Figure 3. It is seen that the test is successfully completed with the robot reaching the goal safely, while avoiding both rocks and the two impassable regions.

## 6 Conclusions

Using Local, Regional, and Global Traversability Indices, three navigational behaviors are developed that together with the goal-seeking behavior comprise the robot navigation strategy. This navigation strategy provides a framework for smooth integration of *sensor-based*, *reactive* behaviors such as Traverse-Local and Traverse-Regional with *map-based*, *deliberative* behaviors such as Traverse-Global and Seek-Goal. This integration is demonstrated in a graphical simulation study.

Terrain-based navigational behaviors based on traversability indices are critical components of any field robot navigation strategy. These behaviors provide a means for incorporating different terrain characteristics into the robot navigation logic. Current research is focused at implementation and field testing of the methodology described in this paper on a Pioneer all-terrain mobile robot.

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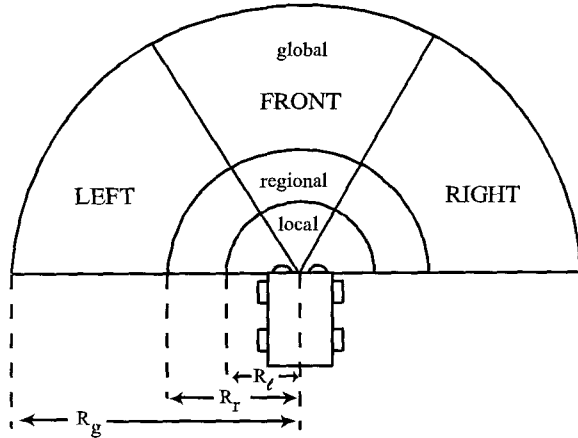


Figure 1. Definition of traversable regions

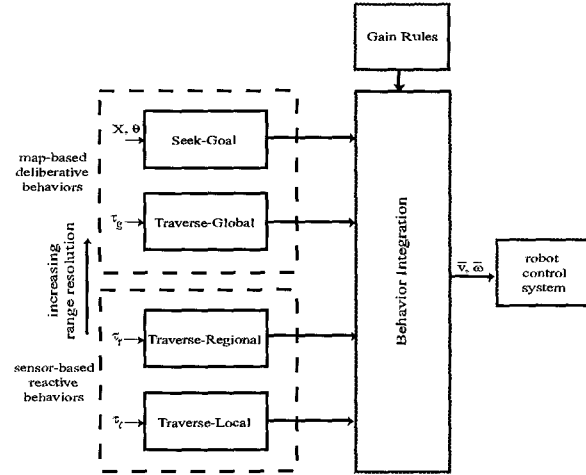


Figure 2. Block diagram of the robot navigation system

The figure displays a 4x4 grid of payoff matrices for a 2-player game. The columns are labeled with player 1's strategy (poor, low, moderate, high) and the rows with player 2's strategy (poor, low, moderate, high). The payoff matrices are labeled with  $\tau_l$  and  $\tau_r$ . The matrices show how payoffs change as the number of players ( $n$ ) increases from 2 to 16.

The payoff matrices are arranged in a 4x4 grid, with the following structure:

- Columns:** poor, low, moderate, high
- Rows:** poor, low, moderate, high
- Payoff Matrices:**
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Table 1. Turn rules for the traverse-terrain behavior  
(R=right turn, L=left turn, O=on-course)

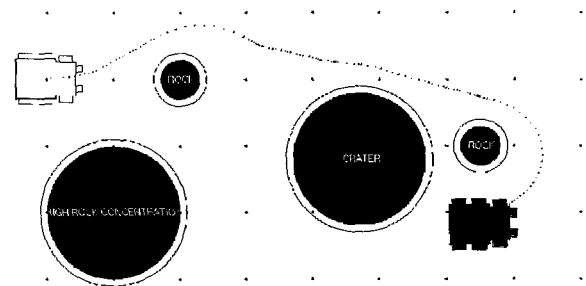


Figure 3: Robot paths using the fuzzy logic navigation strategy